Assume for the present that some positive (or negative) charges are introduced in the interior of a conductor. An electric field will be set up in the conductor, the field exerting a force on the charges and making them move away from one another. This movement will continue until all the charges reach the conductor surface and redistribute themselves in such a way that both the charge and the field inside vanish.

The charge distribution on the surface of a conductor depends on the shape of the surface. Obviously, tangential component of the electric field must be zero.

E field on a conductor surface is everywhere normal to the surface.
2.6 Dielectrics in Static Electric Field

The presence of an external electric field causes a force to be exerted on each charged particle and results in small displacements of positive and negative charges in opposite directions. These displacements polarize a dielectric material and create electric dipoles, which equivalent to an volume charge density $\rho_p$.

The polarized charge density is related to the polarization vector $\mathbf{P}$ by

$$\rho_p = -\nabla \cdot \mathbf{P}$$

The “-” sign means that the field generated by the electric dipoles is in the opposite direction of external electric field $\mathbf{E}$. 

A polarized dielectric medium
The divergence postulation must be modified to include the effect of $\rho_p$

$$\nabla \cdot \vec{E} = \frac{1}{\varepsilon_0} (\rho + \rho_p)$$

or

$$\nabla \cdot (\varepsilon_0 \vec{E} + \vec{P}) = \rho$$

We now define a new fundamental field quantity, the **electric flux density**, or **electric displacement**, $\vec{D}$, such that

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \quad (C / m^2)$$

or

$$\nabla \cdot \vec{D} = \rho \quad (C / m^3)$$
When the dielectric properties of the medium are linear and isotropic, the polarization is directly proportional to the electric field intensity, and the proportionality constant is independent of the direction of the field. We have

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$

where $\chi_e$ is a dimensionless quantity called electric susceptibility. It is obvious that

$$\vec{D} = \varepsilon_0 (1 + \chi_e) \vec{E} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon \vec{E} \ (C / m^2)$$

<table>
<thead>
<tr>
<th>Material</th>
<th>Relative Permittivity, $\varepsilon_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1.0</td>
</tr>
<tr>
<td>Bakelite</td>
<td>5.0</td>
</tr>
<tr>
<td>Glass</td>
<td>4–10</td>
</tr>
<tr>
<td>Mica</td>
<td>6.0</td>
</tr>
<tr>
<td>Oil</td>
<td>2.3</td>
</tr>
<tr>
<td>Paper</td>
<td>2–4</td>
</tr>
<tr>
<td>Parafin wax</td>
<td>2.2</td>
</tr>
<tr>
<td>Plexiglass</td>
<td>3.4</td>
</tr>
<tr>
<td>Polyethylene</td>
<td>2.3</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>2.6</td>
</tr>
<tr>
<td>Porcelain</td>
<td>5.7</td>
</tr>
<tr>
<td>Rubber</td>
<td>2.3–4</td>
</tr>
<tr>
<td>Soil (dry)</td>
<td>3–4</td>
</tr>
<tr>
<td>Teflon</td>
<td>2.1</td>
</tr>
<tr>
<td>Water (distilled)</td>
<td>80</td>
</tr>
<tr>
<td>Seawater</td>
<td>72</td>
</tr>
</tbody>
</table>
2.7 Boundary Conditions for Electrostatic Fields

Electromagnetic problems often involve media with different physical properties and require the knowledge of the relations of the field quantities at an interface between two media. For instance, we may wish to determine how the \( \mathbf{E} \) and \( \mathbf{D} \) vectors change in crossing an interface. We already know the boundary conditions that must be satisfied at a conductor/free space interface. These conditions have been given in previous section. We now consider an interface between two general media shown in the figure.

Let us construct a small path \( abcda \) with sides \( ab \) and \( cd \) in media 1 and 2, respectively, both being parallel to the interface and equal to \( \Delta w \). We apply
\[
\oint_C \mathbf{E} \cdot d\mathbf{l} = 0
\]
to the path. If we let sides \( bc=da= \Delta h \) approach zero, we have
\[
\int_{abcda} \mathbf{E} \cdot d\mathbf{l} = \mathbf{E}_1 \cdot \Delta w + \mathbf{E}_2 \cdot (-\Delta w) = E_{1t} \cdot \Delta w - E_{2t} \cdot \Delta w = 0
\]
Therefore,
\[
E_{1t} = E_{2t} \quad (V/m)
\]
which states that the tangential component of an \( \mathbf{E} \) field is continuous across an interface.
2.7 Boundary Conditions for Electrostatic Fields (cont.)

In order to find a relation between the normal components of the fields at a boundary, we construct a small pillbox with its top face in medium 1 and bottom face in medium 2, as illustrated in the figure. The faces have an area $\Delta S$, and the height of the pillbox $\Delta h$ is vanishingly small. Applying Gauss’s law,

$$\oint_S \bar{E} \cdot d\bar{s} = \frac{Q}{\varepsilon_0} \quad \text{or} \quad \oint_S \bar{D} \cdot d\bar{s} = Q$$

we have

$$\oint_S D \cdot d\bar{s} = (\bar{D}_1 \cdot \hat{a}_{n_2} + \bar{D}_2 \cdot \hat{a}_{n_1}) \Delta S$$

$$= \hat{a}_{n_2} \cdot (\bar{D}_1 - \bar{D}_2) \Delta S = \rho_s \Delta S$$

or

$$D_{1n} - D_{2n} = \rho_s \left( \frac{C}{m^2} \right)$$

which the reference unit normal is outward from medium 2.

To repeat in concise form, the boundary conditions that must be satisfied for static electric fields are as follows:

$$E_{1t} = E_{2t} \quad (V / m)$$

$$D_{1n} - D_{2n} = \rho_s \left( \frac{C}{m^2} \right)$$
2.7 Boundary Conditions for Electrostatic Fields (cont.)

**EXAMPLE 3–15** Two dielectric media with permittivities $\epsilon_1$ and $\epsilon_2$ are separated by a charge-free boundary as shown in Fig. 3–25. The electric field intensity in medium 1 at the point $P_1$ has a magnitude $E_1$ and makes an angle $\alpha_1$ with the normal. Determine the magnitude and direction of the electric field intensity at point $P_2$ in medium 2.

**Solution** Two equations are needed to solve for two unknowns $E_{2t}$ and $E_{2n}$. After $E_{2t}$ and $E_{2n}$ have been found, $E_2$ and $\alpha_2$ will follow directly. Using Eqs. (3–118) and (3–123), we have

$$E_2 \sin \alpha_2 = E_1 \sin \alpha_1$$  \hspace{1cm} (3–127)

$$D_{1n} - D_{2n} = 0$$

$$\epsilon_2 E_2 \cos \alpha_2 = \epsilon_1 E_1 \cos \alpha_1.$$  \hspace{1cm} (3–128)

Division of Eq. (3–127) by Eq. (3–128) gives

$$\frac{\tan \alpha_2}{\tan \alpha_1} = \frac{\epsilon_2}{\epsilon_1}.$$  \hspace{1cm} (3–129)

The magnitude of $E_2$ is

$$E_2 = \sqrt{E_{2t}^2 + E_{2n}^2} = \sqrt{(E_2 \sin \alpha_2)^2 + (E_2 \cos \alpha_2)^2}$$

$$= \left[ (E_1 \sin \alpha_1)^2 + \left( \frac{\epsilon_1}{\epsilon_2} E_1 \cos \alpha_1 \right)^2 \right]^{1/2}$$

or

$$E_2 = E_1 \left[ \sin^2 \alpha_1 + \left( \frac{\epsilon_1}{\epsilon_2} \cos \alpha_1 \right)^2 \right]^{1/2}.$$  \hspace{1cm} (3–130)

By examining Fig. 3–25, can you tell whether $\epsilon_1$ is larger or smaller than $\epsilon_2$?